Slope at the Middle of Three Points

Steve Turley, Aug 20, 2018

Consider the three points , , and . The goal is to estimate the slope of the smooth curve going through these three points at the location of the center point. If the points are equally spaced: then the best solution is the central difference formula.

You can see the order of this approximation by considering the Taylor Series expansion of where about the point .

Substituting Equation  into Equation  with gives the following.

Thus, the error in the approximation for the derivative using this central difference formula for equally spaced points is of second order.

There are a couple of ways to consider symmetric formulas for unequally spaced points which give Equation  for equally spaced points. One is to take the average for the forward and backward formulas for the derivative.

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Where and . From the Taylor series expansion in Equation ,

Thus, the error in this case is proportional to and is linear in which is larger than in the case of symmetrically spaced points. Note that Equation  reduces to Equation  when .

Another way to construct a symmetric difference formula is to find the quadratic function between these three points and then take the slope. If we center the quadratic on the middle point where we want the slope and we let

At the point ,

At the point .

At the point ,

Solving for , we have

With irregularly spaced points with and as before,

Inserting Equation  into Equations  and , then solving the two remaining equations for yields the equation for the slope.